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A Cantilever Beam Method for Evaluating Young's Modulus and Poisson's Ratio of Thermal Spray Coatings

E.F. Rybicki, J.R. Shadley, Y. Xiong, and D.J. Greving

Young's modulus and Poisson's ratio for thermal spray coatings are needed to evaluate properties and characteristics of thermal spray coatings such as residual stresses, fracture toughness, and fatigue crack growth rates. It is difficult to evaluate Young's modulus and Poisson's ratio of thermal spray coatings because coatings are usually thin and attached to a thicker and much stiffer substrate. Under loading, the substrate restricts the coating from deforming. Since coatings are used while bonded to a substrate, it is desirable to have a procedure to evaluate Young's modulus and Poisson's ratio in situ.

The cantilever beam method to evaluate the Young's modulus and Poisson's ratio of thermal spray coatings is presented. The method uses strain gages located on the coating and substrate surfaces. A series of increasing loads is applied to the end of the cantilever beam. The moment at the gaged section is calculated. Using a laminated plate bending theory, the Young's modulus and Poisson's ratio are inferred based on a least squares fit of the equilibrium equations. The method is verified by comparing predicted values of Young's modulus and Poisson's ratio with reference values from a three-dimensional finite element analysis of the thermal spray coated cantilever beam. The sensitivity of the method is examined with respect to the accuracy of measured quantities such as strain gage readings, specimen dimensions, applied bending moment, and substrate mechanical properties. The method is applied to evaluate the Young's modulus and Poisson's ratio of four thermal spray coatings of industrial importance.

1. Introduction

THERMAL spray coatings have found many applications in a variety of industries. Coatings offer low cost, high performance solutions for extending life of metal components in severe environments. Thermal spray coatings are attractive for providing increased wear performance, corrosion resistance, and in the case of thermal barrier coatings, enhanced thermal insulation. Applications of thermal spray coatings include pump impellers and housings, valve stems and seats, sliding mechanisms, thrust bearings, and precision machined parts. Thermal barrier coatings are being used to enhance fuel efficiency for internal combustion engines and gas turbine engines. Specific applications are to diesel engine piston crowns and to gas turbine vanes and shrouds. Thermal spray coatings are also used for repairs. Worn parts can often be built up and then machined to meet size tolerances for less expense than replacement parts.

While thermal spray coatings offer many advantages, some aspects need to be investigated and better understood to improve performance and increase the scope of applications. Two important properties of thermal spray coatings that are not easily determined are Young's modulus and Poisson's ratio. Accurate values of Young's modulus and Poisson's ratio are needed to evaluate residual stresses, bond strength, fatigue crack growth rates, and coating stresses during in-service loading conditions. The thermal spray process inherently introduces residual stresses in the coatings. Residual stresses have been shown to have an important role in the debonding of thermal spray coatings (Ref 1) and the performance of thermal barrier coatings (Ref 2). Thus, an accurate procedure to evaluate residual stresses is important. Techniques to experimentally evaluate residual stresses include x-ray diffraction (Ref 3), hole drilling (Ref 4), the modified layer removal method (Ref 5), and beam curvature (Ref 6). Each of these methods requires values of the Young's modulus, E_c, and Poisson's ratio, ν_c, of the coating.

It is important to have a procedure to evaluate the Young's modulus and Poisson's ratio in situ because all coatings are used while bonded to a substrate. To date, obtaining in situ values of E_c and ν_c has been difficult. No method has achieved industry acceptance. One problem encountered is that the coating is usually thin; therefore, it is difficult to obtain a specimen made entirely of coating material to conduct a modulus test. Another difficulty is that as-sprayed properties are desired. Therefore, it is necessary to use a coated substrate specimen and infer the properties from the composite material behavior.

The purpose of this paper is to present a method for the in situ evaluation of Young's modulus and Poisson's ratio for thermal spray coatings. The method is based on a least squares fit of the equilibrium equations utilizing the strain gage data. For verification, results of the method were compared with Young's modulus and Poisson's ratio values from established solutions. The sensitivity of calculated values of Young's modulus and Poisson's ratio to uncertainties in the measured input quantities was examined. Reasonable estimates of uncertainties in strain readings, dimensions, loadings, and mechanical properties were selected to simulate worst case conditions. Next, the method was applied to four industrially important coatings. Results and the general applicability of the cantilever beam method to thermal spray coatings are discussed.

Keywords cantilever beam method, elastic modulus, mechanical properties, Poisson's ratio

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2. Goals

There are four goals of this work. One goal is to present a procedure for the in situ evaluation of Young's modulus and Poisson's ratio for thermal spray coatings. A test that is easy to conduct and inexpensive is sought. The geometry should be a shape that is easy to machine and spray. The loading should be simple to apply and remain constant while readings are being taken. The user should be able to determine the loading accurately. The method should be able to evaluate Young's modulus and Poisson's ratio of the coating for both tension and compression loadings. The second goal is to present a verification of the method. The third goal is to examine how sensitive the calculated values of Young's modulus and Poisson's ratio are to uncertainties in the measured quantities that are inputs to the analysis method. The fourth goal is to apply the method to thermal spray coatings of industrial importance.

3. Approach

The approach to accomplishing these goals is to select a basic configuration, such as a thermal spray coated flat strip, that can be loaded as a cantilever beam. A simple loading is applied by hanging weights from the end of the cantilever beam. Strain gages are placed on the coating and the substrate surfaces. An analysis of the bending moments and forces applied to the gaged section leads to four equations containing the unknown Young's modulus and Poisson's ratio of the coating. These four equations are solved in a least squares manner to evaluate the Young's modulus and Poisson's ratio of the coating. Details of the specimen geometry, the loading, and the analysis procedure are described in the following sections.

3.1 *Experimental Equipment*

Figure 1 illustrates the experimental equipment used for determining Young's modulus and Poisson's ratio of the coating. A thermal spray coating is applied to a beam with dimensions shown in Fig. 2. Four resistance type biaxial strain gages are bonded to the beam. Two biaxial strain gages are placed on the coated surface, and two are placed directly opposite on the substrate side. Figure 2 shows the location of the four strain gages. The instrumented beam is then secured in a vise and connected to a multichannel strain indicator. A force is applied to the end of the beam through calibrated weights hung via a wire and weight pan as shown in Fig. 1. The known moment and measured strain are recorded on a data sheet and are used as input to a computer program for calculating Young's modulus and Poisson's ratio of the coating. There are approximately thirty biaxial strain readings recorded for each test, and end loads are typically applied in 3 to 5 N increments until a maximum end load of 25 N is reached.

3.2 *Analysis of Strain Gage Data Using the Laminated Plate Theory*

The laminated plate theory is used in the cantilever beam method to relate the unknown Young's modulus and Poisson's ratio of the coating to the loadings at the gaged sections. It is assumed that the coating and substrate are isotropic materials. Four equations result in terms of the two unknown quantities, E_c and ν_c. If all dimensions, forces, and properties were known exactly, then any two of the four equations could be used to solve for Young's modulus and Poisson's ratio. However, measured quantities, loads, and properties are not known exactly. Therefore, a least squares method is used to evaluate Young's modulus and Poisson's ratio of the thermal spray coating. The laminated plate theory assumes a linear strain distribution through the thickness of the coated cantilever beam and plane stress conditions. The four equilibrium equations are two moment equations and two force equations. The least squares method minimizes a function composed of these four equations, which includes the coating properties. The "Appendix" of this paper contains details of equations used for the laminated plate theory and least squares calculations.

4. Verification Analyses

A three-dimensional finite element analysis was used to verify the cantilever beam method. A finite element model of the test specimen was constructed with specified coating modulus and Poisson's ratio and dimensions used in the experimental procedure. An end force was applied and strains were calculated from the finite element analysis at the gage

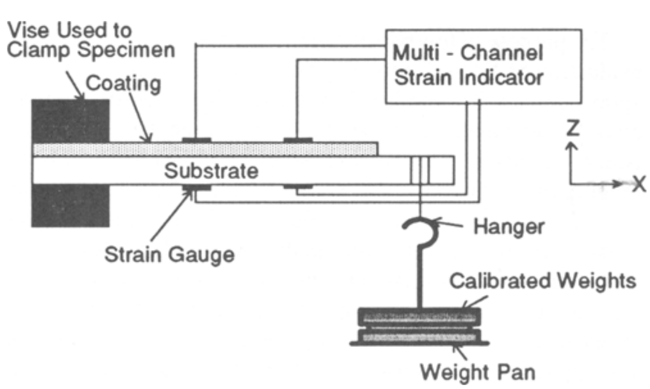

Fig. 1 Experimental equipment and coated beam for the cantilever beam method

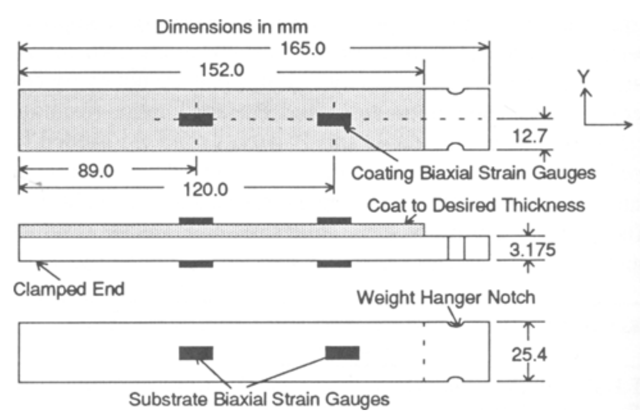

Fig. 2 Test specimen with dimensions and strain gage locations

locations. The known moment and strains were treated as experimental data and used as input for the cantilever beam method calculations. This procedure was performed for two coatings. One was a coating expected to exhibit a high modulus (i.e., a ceramic material), and the other was a coating with a low modulus (i.e., a metal). Table 1 shows the Young's moduli and Poisson's ratios used in the finite element analysis and the values calculated using the cantilever beam method. The tungsten carbide coating is an example of a HVOF process, and the nickel aluminum coating is an example of an arc spray process.

Results from the verification analysis show the cantilever beam method as an accurate experimental method.

5. Sensitivity Analysis

When the analysis procedure is used to evaluate Young's modulus and Poisson's ratio of a thermal spray coating, it is presumed that all dimensions and other measured properties characterizing the specimen and test conditions are known precisely. These quantities are the strain gage readings on the surface of the coating and the substrate, the dimensions of the specimen, the mechanical properties of the substrate, and the bending moment acting on the gaged section. In reality, measurements of these quantities contain some degree of uncertainty. Therefore, it is important to determine how uncertainties in the input data affect the calculated values of Young's modulus and Poisson's ratio of the coating using the cantilever beam method.

The sensitivity analysis has three principal purposes. One purpose is to show how the calculated values of Young's modulus and Poisson's ratio of the coating are changed if one of the measured inputs is different from the exact value. For example, the accuracy of a strain gage reading can be within 1 $\mu\varepsilon$. The sensitivity analysis determines the error in the calculated values of Young's modulus and Poisson's ratio of the coating if a strain gage reading is 1 $\mu\varepsilon$ more or less than the exact value. The second purpose of the sensitivity analysis is to rank the measured quantities in terms of which quantities should be measured more precisely. For example, the sensitivity analysis will show which measured quantities, if any, are likely to cause a difference in the calculated Young's modulus of the coating of more than 10% and which measured quantities are likely to cause a difference in Young's modulus of the coating that is less than 1%. On the basis of these results, special procedures could be used, if needed, to reduce the uncertainties in measuring the dimensions and properties causing a 10% difference in the calculated Young's modulus of the coating. The third purpose of the sensitivity analysis is to provide an indication of the robustness or stability of the cantilever beam method for evaluating Young's modulus and Poisson's ratio of the coating.

In the following, a specimen size and coating thickness are chosen, and a reasonable range of uncertainty for each of the measured quantities is estimated. The cantilever beam method is applied to determine how uncertainties in each of the measured parameters alter the calculated values of Young's modulus and Poisson's ratio. Answers are given in terms of a percent of the exact values of Young's modulus and Poisson's ratio.

5.1 *Dimensions, Mechanical Properties, and Loadings for the Sensitivity Analysis*

The sensitivity analysis was conducted for the specimen size recommended in this paper. The specimen width was 25.4 mm, and the length was 152 mm. The thicknesses of the substrate and the coating were 3.175 mm and 1.143 mm, respectively. Two substrate materials, aluminum and steel, were selected to represent a large range of values of the substrate modulus. Four values of coating modulus and three Poisson's ratios were chosen to represent a range of coating material properties of interest. The four values of coating modulus were 68.9, 137.9, 206.8, and 275.8 GPa to represent metal coatings. Three Poisson's ratios of 0.1, 0.2, and 0.3 were selected for each modulus for a total of twelve hypothetical coating materials for the sensitivity analysis. A combination of the 12 coating materials and the 2 substrate materials results in a total of 24 hypothetical coated specimens for the sensitivity analysis.

All of the analyses performed for the sensitivity analysis were assessed using a computational stress analysis model rather than experiments. The stress analysis (based on the laminated plate theory, Ref 7) was used to calculate the "exact" strains at the strain gaged section. The "exact" strains, dimensions, and mechanical properties and the uncertainties for each measured quantity were inputs to 240 applications of the cantilever beam method to determine how sensitive the calculated values of Young's modulus and Poisson's ratio of the coating are to the uncertainties in measured values. In each case of the sensitivity analysis, the loading at the gaged section was selected so that the strain in the *X*-direction, shown in Fig. 1 and 2, was 100 $\mu\varepsilon$.

5.2 *Accuracy of Measured Dimensions and Mechanical Properties*

Ten dimensions and mechanical properties were varied for the sensitivity analysis. These are the four strain gage readings, the thicknesses of the coating and the substrate, the width of the specimen, the Young's modulus and Poisson's ratio of the substrate, and the applied moment at the gaged section. Table 2 lists the variables and uncertainties used in the sensitivity analysis.

Table 1 Comparison of cantilever beam method to three-dimensional finite element analysis

Coating	Finite element model		Cantilever beam method		Difference	
type	E_c, GPa	ν_c	E_c, GPa	ν_c	E_c, GPa	ν_c
Nickel aluminum	83.944	0.193	84 116	0.190	0.21%	1.30%
Tungsten carbide	176.506	0.290	177.195	0.290	0.39%	0.0%

5.3 *Sensitivity Analysis Results*

The sensitivity analysis consists of a series of analyses of the cantilever beam method in which each of the parameters is varied by the amount listed in Table 2. For each sensitivity analysis, the percent change in Young's modulus and Poisson's ratio was evaluated for the uncertainty in one parameter. A summary of the sensitivities of the calculated values of Young's modulus and Poisson's ratio for the twelve hypothetical coating materials is presented in Table 3 for the aluminum substrate.

The sensitivity analysis shows that uncertainties in measured quantities of the magnitudes shown in Table 2 affect the calculated values of Young's modulus by 2% or less. The gage on the coating in the *X*-direction has the largest effect on the calculated value of Young's modulus. Note that the 2% uncertainty in Young's modulus shown for ε_{XCG} in line 1 of Table 3 is evaluated for ε_{XCG}= 100 με. If the loading causes ε_{XCG} to be 400 με, then the corresponding uncertainty in the modulus due to 1 με uncertainty in the strain, ε_{XCG}, would be one fourth of the value shown in line 1 of Table 3 or ≤0.5%. The same ratio of ε_{XCG} compared to 100 με applies to the percent uncertainty in E_c for lines 2, 3, and 4 in Table 3.

Table 2 Variables and uncertainties used in sensitivity analysis

Symbol	Description	Maximum uncertainty
ε_{XCG}	Longitudinal strain gage reading on coating	±1 με
ε_{YCG}	Transverse strain gage reading on coating	±1 με
ε_{XSG}	Longitudinal strain gage reading on substrate	±1 με
ε_{YSG}	Transverse strain gage reading on substrate	±1 με
H	Thickness of substrate	±0.0127 mm
T_c	Thickness of coating	±0.0254 mm
b	Width of specimen	±0.0127 mm
E_s	Young's modulus of the substrate	±2% of E_s
ν_s	Poisson's ratio of the substrate	±5% of ν_s
$\overline{M}$	Bending moment to cause ε_{XCG} to be 100 με	±0.1% of M

Table 3 Percentage uncertainty of coating Young's modulus and Poisson's ratio due to uncertainties in measured quantities—aluminum substrate (loading causes ε_{XCG} to be 100 με

Measured quantity	Uncertainty in measured quantity	Resulting uncertainty in E_c, %	Resulting uncertainty in ν_c, %
ε_{XCG}	1 με	≤2.0	≤5.0
ε_{YCG}	1 με	≤0.3	5.0-21.0(a)
ε_{XSG}	1 με	≤1.1	≤4.0
ε_{YSG}	1 με	≤0.3	2.0-11.0(a)
H	0.0127 mm	≤0.6	≤0.8
T_c	0.0254 mm	≤1.2	≤2.2
b	0.0127 mm	≈0.0	≤0.1
E_s	2%	≤0.5	≤0.2
ν_s	5%	≤0.2	≤4.0
$\overline{M}$	0.1%	≤0.1	≤0.1

(a) These uncertainties have an upper and lower bound due to the wide range of error resulting from coatings with different values of Young's modulus and Poisson's ratio.

As might be expected, the next most important measurement to obtain accurately is the strain gage reading in the *X*-direction on the substrate. However, an uncertainty in the reading of 1 με causes an uncertainty in the Young's modulus of 1.1% or less for an ε_{XCG} reading of 100 με. Uncertainties in other measurements, such as the thicknesses of the substrate and the coating, the width of the specimen, and the Young's modulus and Poisson's ratio of the substrate, and the applied moment lead to uncertainties in Young's modulus that are less than 0.6%.

Assume that, for any set of measurements, not all measured quantity errors would be the maximum values indicated in Table 2 and in directions that continually add uncertainties. Uncertainty in the computed values of E_c and ν_c can be estimated as (Ref 8):

$$w_c = [w_1^2 + w_2^2 + \ldots + w_n^2]^{1/2} \quad \text{(Eq 1)}$$

where w_c is the uncertainty in the computed value (E_c or ν_c) due to all measured quantity uncertainties, and w_1, w_2, ..., w_n are the uncertainties in the computed value due to each measured quantity uncertainty acting separately (all other measured quantities being exact). Using Eq 1, the uncertainty in E_c for the aluminum substrate due to all measured quantity uncertainties is estimated to be less than 2.6%.

The calculated values of Poisson's ratio are more sensitive to uncertainties in the measured quantities than are the calculated values of Young's modulus. This is shown in Table 3 for the aluminum substrate. The highest percent uncertainty occurs for the cases where the Poisson's ratio of the coating is different from the Poisson's ratio of the substrate. The Poisson's ratio of the coating is most sensitive to uncertainties in the transverse strain gage readings on the coating and the substrate.

Table 4 shows coating Young's modulus and Poisson's ratio uncertainties for the steel substrate. These uncertainties are generally higher than the uncertainties for the aluminum substrate shown in Table 3. Again, it is most important to obtain an accurate reading from the strain gage on the coating surface in the *X*-direction or bending direction. The next most important gage

Table 4 Percentage uncertainty of coating Young's modulus and Poisson's ratio due to uncertainties in measured quantities—steel substrate (loading causes ε_{XCG} to be 100 με

Measured quantity	Uncertainty in measured quantity	Resulting uncertainty in E_c, %	Resulting uncertainty in ν_c, %
ε_{XCG}	1 με	≤3.5	2.0-13.0(a)
ε_{YCG}	1 με	≤1.1	6.0-38.0(a)
ε_{XSG}	1 με	≤2.8	1.0-12.0(a)
ε_{YSG}	1 με	≤0.9	3.0-25.0
H	0.0127 mm	≤1.2	≤2.4
T_c	0.0254 mm	≤1.2	≤4.4
b	0.0127 mm	≤0.1	≤0.1
E_s	2%	≤0.1	≤4.0
ν_s	5%	≤1.0	1.0-10.0(a)
$\overline{M}$	0.1%	≤0.2	≤0.2

(a) These uncertainties have an upper and lower bound due to the wide range of error resulting from coatings with different values of Young's modulus and Poisson's ratio.

reading is from the strain gage on the substrate in the bending direction. Errors in the thickness of the specimen have a slightly higher effect on the uncertainty of the Young's modulus than errors in the strain gage readings in the transverse or *Y*-direction. The largest percent uncertainties occur for the cases of low coating modulus and low Poisson's ratio.

When Eq 1 is applied to the data in Table 4, the uncertainty in E_C for the steel substrate due to all measured quantity uncertainties is estimated to be less than 5.1% for a loading that produces a strain in ε_{XCG} of 100 $\mu\varepsilon$.

5.4 *Results of Sensitivity Analyses*

Based on the specimen dimensions, mechanical properties, and uncertainties in input parameters for the least squares method, the following results are presented for the cantilever beam method.

- To reduce the effect of uncertainty in strain gage readings, the applied bending moment on the gaged section should be as large as possible without failing the coating or causing nonlinear behavior.
- The Young's modulus of the coating is most sensitive to uncertainties in readings of the strain gage on the coating in the bending direction.
- The calculated value of Poisson's ratio of the coating is most sensitive to uncertainties in readings of the transverse strain gage on the coating.
- If the coating Young's modulus and Poisson's ratio are independent of the type of substrate material, then accuracy is enhanced if the substrate with the lowest modulus is used for the test specimen.
- The calculated Young's modulus of the coating is sensitive to uncertainties in substrate thickness, the coating thickness, and Young's modulus of the substrate.
- Uncertainties in the width of the specimen, the applied bending moment, and the Poisson's ratio of the substrate have smaller effects on the Young's modulus of the coating.
- The calculated Poisson's ratio was more sensitive to uncertainties in Young's modulus and Poisson's ratio of the substrate for the steel substrate.
- Uncertainties in the Young's modulus for coatings tested on an aluminum substrate are less than 2.6%, and on a steel substrate, they are less than 5.1% if the strain reading in the bending direction on the coating is 100 $\mu\varepsilon$ or more.

6. Application to Coatings of Industrial Importance

The cantilever beam method was used to determine the Young's modulus and Poisson's ratio of four thermal spray coatings. A coating of TAFA Bondarc 75B nickel 5 aluminum (Hobart TAFA Inc., Concord, NH) was applied by a model 9000 wire arc spray system (Hobart TAFA Inc.) to a 6061 aluminum substrate. A Walcoloy #5 (Wall Colmonoy Corp., Madison Heights, MI) austenitic stainless steel coating was applied by a Wall Colmonoy model WG-500 wirespray system to an AISI 1018 steel substrate. A JK117 (Stellite, Goshen, IN) coating of 83% tungsten carbide and 17% cobalt was applied by Jet Kote II (Stellite) to an AISI 1018 steel substrate. A plasma sprayed 8% yttria stabilized zirconia was applied to a 6061 aluminum substrate. Values of Young's modulus and Poisson's ratio calculated by the cantilever beam method for these coatings are listed in Table 5.

Young's modulus and Poisson's ratio values from Ref 5 and 10 are reported in Table 6. Table 6 also lists values found by the cantilever beam method. Results from Table 6 for the cantilever beam method are similar to results by other methods.

7. Summary and Conclusions

Two important mechanical properties of thermal spray coatings that are not easily evaluated are Young's modulus and Poisson's ratio. These properties are needed to evaluate residual stresses, fracture and fatigue properties, and in-service stresses and strains. The cantilever beam method was developed to determine in situ values of Young's modulus and Poisson's ratio of thermal spray coatings. The method involves a coated substrate "beam" that is 25.4 mm wide, 152 mm long, and 3.175 mm thick. The beam is clamped in a vise at one end, and weights are applied to the other end. Strain gages are placed on the coating

Table 5 Young's modulus and Poisson's ratio values of thermal spray coatings determined by the cantilever beam method

Coating material	Application process	Young's modulus, GPa	Poisson's ratio
Tungsten carbide/cobalt-83% WC-17% Co	HVOF	179.95	0.278
Zirconia-8% yttria stabilized-ZrO_2	Plasma	45.51	0.181
Nickel aluminum-95% Ni 5% Al	Wire arc spray	81.36	0.163
Stainless steel-austenitic	Wire flame spray	59.78	0.151

Table 6 Young's modulus and Poisson's ratio values for different methods

Coating material	Method of evaluation	Young's modulus, GPa	Poisson's ratio
APS/83% WC-17% Co (Ref 9)	Ultrasonic	150	0.2
HVOF/83% WC-17% Co	Cantilever beam method	179.95	0.278
Plasma spray/ZrO_2-7% yttria stabilized (Ref 10)	Uniaxial tension	40	0.2
Plasma spray/ZrO_2-8% yttria stabilized	Cantilever beam method	45.51	0.181

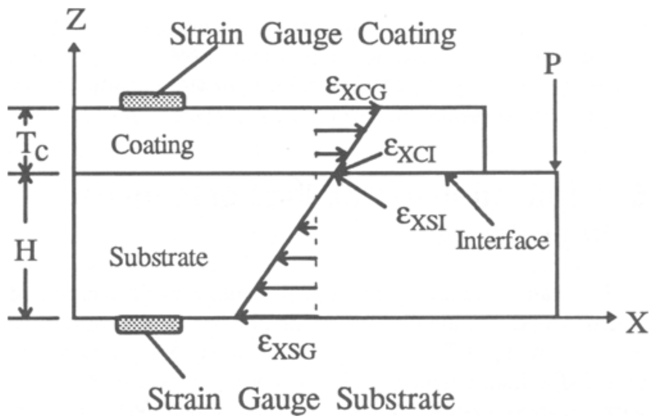

Fig. 3 Strain distribution for coated cantilever beam with applied load P

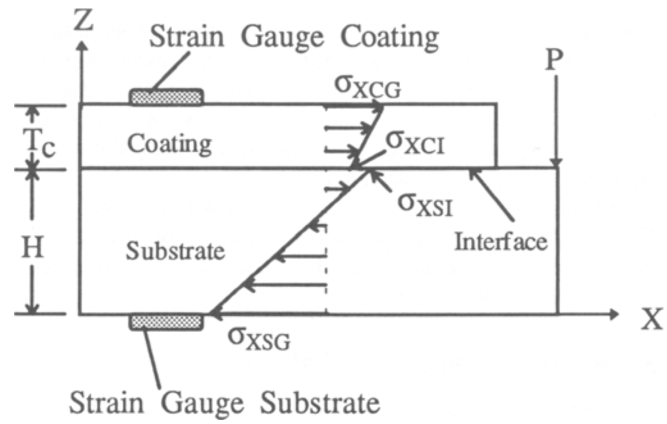

Fig. 4 Stress distribution for coated cantilever beam with applied load P

and substrate surfaces. Readings are taken as the weights are applied. Inputs to the analysis to evaluate the Young's modulus and Poisson's ratio of the coating are strain gage data, the dimensions of the specimen, properties of the substrate, and the applied bending moment. The analysis method uses the equations of equilibrium for bending moments and forces at the gaged section. A verification of the method was done using a three-dimensional finite element analysis of the thermal spray coated test specimen with the applied loading.

The cantilever beam method has several features that make it attractive. The method is easy to use and inexpensive. The equipment needed is a vise or clamping fixture, strain gages, a strain indicator, a micrometer, a ruler, a hanger, and a set of weights. The specimen is easy to machine and spray. The loading is easy to apply and remains constant during readings. The method can be used to evaluate Young's modulus and Poisson's ratio in tension or compression.

A sensitivity analysis showed that errors in the measurements of dimensions, strain gage readings, and substrate properties needed for input to the method can alter the values of Young's modulus by less than 3%. The values of Poisson's ratio obtained by the method are more sensitive to errors in the measurements needed for input but are sufficiently accurate for applications of practical interest.

The cantilever beam method was applied to four types of coatings used frequently in industrial applications. Values of E_c and ν_c for two of the four coatings were available in the literature and were comparable to the values obtained by the cantilever beam method.

Acknowledgments

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Appendix

The laminated plate theory is used in the cantilever beam method to relate the unknown Young's modulus and Poisson's ratio of the coating to the loadings at the gaged sections. The laminated plate theory assumes a linear strain distribution through the thickness of the coated cantilever beam and plane stress conditions. Figures 3 and 4 show a schematic of the strain and stress distribution, respectively, for the coated cantilever beam with the applied loading. Not shown in Fig. 3 and 4 is b, the width of the specimen, or the strain and stress in the Y-direction. The difference in mechanical properties between the coating and substrate introduces the stress discontinuity at the interface.

The equilibrium equations for the coated beam are as follows where the stresses are related to the forces and moments by:

$$0 = F_X = \int\int \sigma_X \, dzdy \qquad \overline{M}_X = M_X = \int\int \sigma_z \, zdzdy$$

$$0 = F_Y = \int\int \sigma_Y \, dzdy \qquad 0 = M_Y = \int\int \sigma_Y \, zdzdy \qquad \text{(Eq 2)}$$

For the coated beam,

$$F_X = b(\sigma_{XSI} + \sigma_{XSG})\frac{H}{2} + b(\sigma_{XCI} + \sigma_{XCG})\frac{T_c}{2} \qquad \text{(Eq 3)}$$

$$M_X = b\left[\sigma_{XSG}\frac{H^2}{2} + (\sigma_{XSI} - \sigma_{YSG})\frac{H^2}{3}\right] + b\left[\sigma_{XCI}\,T_c\left(\frac{H}{2}+\frac{T_c}{6}\right) + \sigma_{XCG}\,T_c\left(\frac{H}{2}+\frac{T_c}{3}\right)\right] \qquad \text{(Eq 4)}$$

$$F_Y = b(\sigma_{YSI} + \sigma_{YSG})\frac{H}{2} + b(\sigma_{YCI} + \sigma_{YCG})\frac{T_c}{2} \qquad \text{(Eq 5)}$$

$$M_Y = b\left[\sigma_{YSG}\frac{H^2}{2} + (\sigma_{YSI} - \sigma_{YSG})\frac{H^2}{3}\right] + b\left[\sigma_{YCL}\,T_c\left(\frac{H}{2}+\frac{T_c}{6}\right) + \sigma_{YCG}\,T_c\left(\frac{H}{2}+\frac{T_c}{3}\right)\right] \qquad \text{(Eq 6)}$$

The surface stresses, σ_{CG} and σ_{SG}, are related to the strains and the mechanical properties of the coating and substrate from the following:

$$\sigma_{XCG} = \frac{E_c}{(1-\nu_c{}^2)}(\varepsilon_{XCG} + \nu_c\varepsilon_{YCG}) \qquad \text{(Eq 7)}$$

$$\sigma_{XSG} = \frac{E_s}{(1 - \nu_s{}^2)}(\varepsilon_{XSG} + \nu_s \varepsilon_{YSG}) \quad \text{(Eq 8)}$$

where E_c and ν_c are the Young's modulus and Poisson's ratio for the coating and E_s and ν_s are the substrate properties that are known. The surface strains, ε_{XCG}, ε_{YCG}, ε_{XSG}, and ε_{YSG}, are measured with strain gages.

The interface stresses can be calculated from:

$$\sigma_{XCI} = \frac{E_c}{1 - \nu_c^2}(\varepsilon_{XCI} + \nu_c \varepsilon_{YCI}) \quad \text{(Eq 9)}$$

$$\sigma_{XSI} = \frac{E_s}{1 - \nu_s^2}(\varepsilon_{XSI} + \nu_s \varepsilon_{YSI}) \quad \text{(Eq 10)}$$

where the interface strains, ε_{XCI}, ε_{YCI}, ε_{XSI}, and ε_{YSI}, can be found from the assumption of a linear strain dis1tribution for the surface strains.

If all dimensions, forces, and properties were known exactly, then any two of the four equilibrium equations could be used to solve for Young's modulus and Poisson's ratio of the coating. However, measured quantities, loads, and properties are not known exactly. Therefore, a least squares method is used to evaluate Young's modulus and Poisson's ratio of the thermal spray coating. The least squares method minimizes a function composed of the four equilibrium equations and contains Φ, given by Eq 11.

$$\Phi(E_c, \nu_c) = \frac{(H + T_c)^2}{36}[F_X^2 + F_Y^2] + [M_X - \overline{M}_X] + [M_Y]^2 \quad \text{(Eq 11)}$$

The function, $\Phi(E_c, \nu_c)$, is based on minimizing the maximum stress difference. Values for F_X, F_Y, M_X, and M_Y can be found from Eq 3 to 6, and $\overline{M}_X$ is an applied load, which is equal to the applied force, P, times the distance between the load location and the gage location. In order to determine the values of E_c and ν_c that minimize the function $\Phi(E_c, \nu_c)$ for any set of test data, a computer program was developed. In the program, the value of Φ is computed over a range of Young's modulus and Poisson's ratio values. From the program output, the minimum value of Φ is identified, and the associated values of E_c and ν_c are taken as the estimates of the Young's modulus and Poisson's ratio of the coating.

Nomenclature	
ε_{XCG}	Longitudinal strain gage reading on coating
ε_{YCG}	Transverse strain gage reading on coating
ε_{XSG}	Longitudinal strain gage reading on substrate
ε_{YSG}	Transverse strain gage reading on substrate
ε_{XCI}	Longitudinal strain at coating interface
ε_{YCI}	Transverse strain at coating interface
ε_{XSI}	Longitudinal strain at substrate interface
ε_{YSI}	Transverse strain at substrate interface
σ_{XCG}	Longitudinal stress on coating surface
σ_{YCG}	Transverse stress on coating surface
σ_{XSG}	Longitudinal stress on substrate surface
σ_{YSG}	Transverse stress on substrate surface
σ_{XCI}	Longitudinal stress at coating interface
σ_{YCI}	Transverse stress at coating interface
σ_{XSI}	Longitudinal stress at substrate interface
σ_{YSI}	Transverse stress at substrate interface
H	Thickness of substrate
T_c	Thickness of coating
b	Width of the specimen
E_c	Young's modulus of the coating
ν_c	Poisson's ratio of the coating
E_s	Young's modulus of the substrate
ν_s	Poisson's ratio of the substrate
P	Applied load
$\overline{M}$	Applied bending moment at gaged locations

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